

## 11.1

# Up and Down or Down and Up

## Exploring Quadratic Functions

### LEARNING GOALS

In this lesson, you will:

- Model real-world problems using quadratic functions.
- Analyze tables, graphs, and equations for quadratic functions.
- Use the Distributive Property to write a quadratic equation in standard form.
- Compare graphs of quadratic functions.
- Use a graphing calculator to determine the absolute minimum or absolute maximum of a quadratic function.

### KEY TERMS

- standard form (general form) of a quadratic function
- parabola

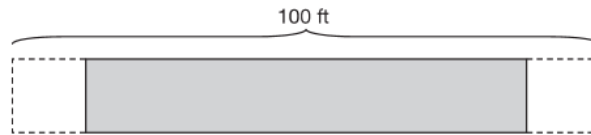
**T**ake a necklace and hold it at both ends with your fingers. What function would you say describes the shape of the necklace? Quadratic? Wrong. This shape is formed by a catenary function.

From bridges to spider webs to architectural arches, catenaries are all around us—and they look so much like parabolas, it's easy to be fooled.

In future math classes—distant future—you'll learn about the differences between catenary functions and quadratic functions. For now, let's just focus on quadratics.

**PROBLEM 1** Who Put the Dogs In?

A dog trainer is fencing in an enclosure, represented by the shaded region in the diagram. The trainer will also have two square-shaped storage units on either side of the enclosure to store equipment and other materials. She can make the enclosure and storage units as wide as she wants, but she can't exceed 100 feet in total length.



1. Let  $s$  represent a side length, in feet, of one of the square storage units.
  - a. Write an expression to represent the width of the enclosure. Label the width in the diagram. Explain your reasoning.

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- b. Write an expression to represent the length of the enclosure. Label the length in the diagram. Explain your reasoning.



- c. Write an expression to represent the area of the enclosure. Explain your reasoning.

You know that you can use the Distributive Property to rewrite a mathematical expression.

For example, to rewrite  $2(5 - 4)$ , you can distribute the 2:

$$2(5 - 4) = 2(5) - 2(4)$$

You can also distribute variables:

$$n(4 - 3n) = n(4) - n(3n)$$

$$= 4n - 3n^2$$

The expression  $n \cdot 3n$  is equivalent to  $n \cdot 3 \cdot n$ , which is the same as  $n \cdot n \cdot 3$ , or  $n^2 \cdot 3$ , or  $3n^2$ .

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2. Use the Distributive Property to rewrite the expression representing the area of the enclosure. Show your work.

The expression you wrote is a quadratic expression. Recall that a quadratic function written in the form  $f(x) = ax^2 + bx + c$ , where  $a \neq 0$ , is in **standard form**, or **general form**. In this form,  $a$  and  $b$  represent numerical coefficients and  $c$  represents a constant.



3. Write the area of the enclosure as a function,  $A(s)$ , in standard form. Then, identify  $a$ ,  $b$ , and  $c$  for the function.



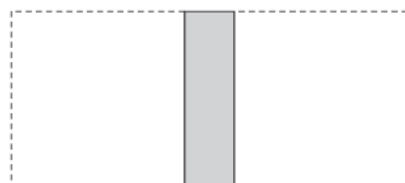
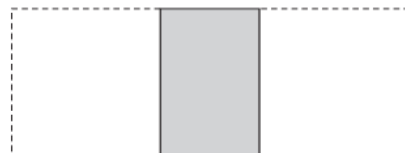
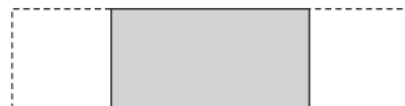
4. Identify the independent and dependent quantities and their units in this problem situation.



The progression of diagrams at the right shows how the area of the enclosure,  $A(s)$ , changes as the side length  $s$  of each square storage unit increases.

5. Describe how the area of the enclosure changes.

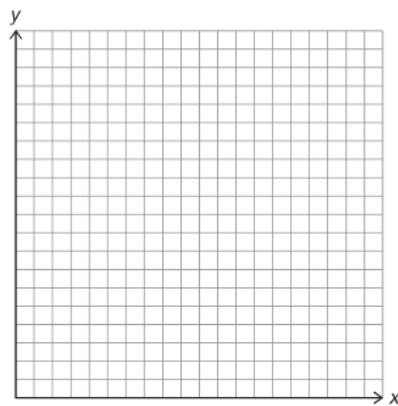
6. Predict what the graph of the function will look like. Explain your reasoning.



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7. Use a graphing calculator to graph the function you wrote in Question 3. Then sketch the graph and label the axes. How can you tell the graph is a quadratic function?



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The shape that a quadratic function forms when graphed is called a **parabola**.

8. Think about the possible areas of the enclosure. Is there a maximum area that the enclosure can contain? Explain your reasoning in terms of the graph and in terms of the problem situation.



You can use a graphing calculator to identify the absolute maximum of a quadratic function.

**Step 1:** Enter the function and press **GRAPH**.

**Step 2:** Press **2ND** and then **CALC**.  
Select **4: maximum**.

**Step 3:** Move your cursor to any point on the curve where the graph is increasing and press **ENTER**.

**Step 4:** Move your cursor to a point directly opposite the point in Step 3 on the curve where the graph is decreasing and press **ENTER**. Then press **ENTER** one more time.

A graph is increasing when it is going up from left to right. It is decreasing when it is going down from left to right.

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9. Use a graphing calculator to determine the absolute maximum of  $A(s)$ . Then describe what the  $x$ - and  $y$ -coordinates of the absolute maximum represent in this problem situation.



10. Determine the dimensions of the enclosure that will provide the maximum area. Show your work and explain your reasoning.

**PROBLEM 2 Handshake Problem**

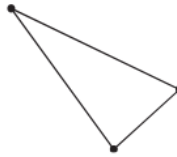
Suppose that there is a monthly meeting at CIA headquarters for all employees. How many handshakes will it take for every employee at the meeting to shake the hand of every other employee at the meeting once?

1. Use the figures shown to determine the number of handshakes that will occur between 2 employees, 3 employees, and 4 employees.

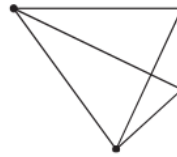
2 employees



3 employees



4 employees

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2. Draw figures to represent the number of handshakes that occur between 5 employees, 6 employees, and 7 employees and determine the number of handshakes that will occur in each situation.

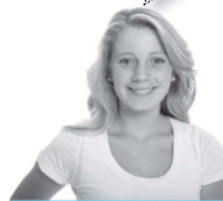
3. Complete the table to record your results.

<b>Number of Employees</b>	2	3	4	5	6	7	$n$
<b>Number of Handshakes</b>							

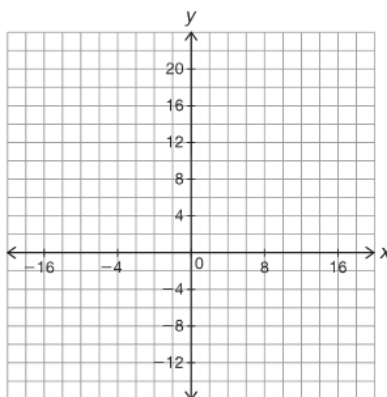
4. Write an expression in the table to represent the number of handshakes given any number of employees.

5. Rewrite the expression representing the handshake pattern as the quadratic function  $H(n)$  in standard form. Identify the independent and dependent quantities.

Look at the relationship between the number of consecutive employees and the number of handshakes. Hint:  $(2 \times 3) \div 2 = 3$ .



6. Graph this function on a graphing calculator. Sketch the graph on the coordinate plane and label the axes.



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7. Determine the absolute minimum of  $H(n)$ . Then, describe what the  $x$ - and  $y$ -coordinates of this point represent in this problem situation.

## Talk the Talk

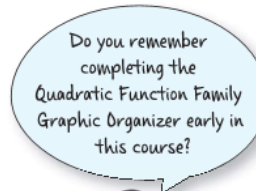


Let's compare the two functions you graphed in this lesson.  
Graph both functions together on a graphing calculator.

Area of dog enclosure:  $A(s) = -2s^2 + 100s$

Handshake Pattern:  $H(n) = \frac{1}{2}n^2 - \frac{1}{2}n$

1. Describe the domain in terms of the function and each problem situation.



2. How can you determine whether the graph of a quadratic function opens up or down based on the equation?

Experiment with your graphing calculator to help you explain your reasoning.



3. How can you determine whether the graph of a quadratic function has an absolute minimum or an absolute maximum?



Be prepared to share your solutions and methods.